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REVIEW

Nonlinear Waves. Edited by S. Leibovich and A. R. Seebass. Cornell University Press, 1974. 331 pp. £9.10.

The subject of nonlinear waves has received enormous attention over the last ten years. Much of the work has concentrated on generalizing ideas about linear dispersive systems to nonlinear ones. Parallel to this, a very important equation in one-dimensional space and time which arises in plasma dynamics and water waves, the Korteweg–de Vries equation, has been essentially solved exactly. The methods used to accomplish this have turned out to have significance for the general theory of equations of evolution, which has been under development at the same time. The present volume is a supplemented version of a series of seminars presented at Cornell University in 1969. The various chapters are the contributions of the different speakers and the whole collection has been ably edited to provide coherence.

A crude classification of the chapters might be (i) basically linear, (ii) weakly nonlinear and (iii) essentially nonlinear. Under the heading (i) there is an introductory chapter by W. D. Hayes and a chapter by S. A. Thau on linear dispersive waves. Thau's chapter contains a nice discussion of the caustic in one dimension. The chapter by C. S. Yih on wave motion in stratified fluids is devoted mainly to the linear theory with a fairly abstract discussion of modes and stability.

Category (ii) contains an interesting chapter by O. M. Phillips which summarizes the theory of wave interactions where combinations of wavenumber and frequency produce resonant overtones and undertones. Laboratory experiments on water waves confirm the basic ideas. The principal chapters in category (iii) are those by G. B. Whitham and R. Miura. Whitham's chapter summarizes his very important work on nonlinear dispersive waves and variational principles. The use of the averaged Lagrangian and its variation provides a framework for equations describing changes in nonlinear wave trains propagating in one dimension. The equations relate wave amplitude, frequency and wavenumber. A basic equation is identified as conservation of 'wave action' and a perturbation procedure is sketched which clarifies the intuitive ideas. W. D. Hayes is also the author of a chapter on conservation of wave action which gives a slightly different version of Whitham's ideas by averaging over the phase shift. Miura's chapter summarizes the very beautiful work of a large group whose main members are M. Kruskal, N. Zabusky, C. Gardner and P. Lax on the Korteweg–de Vries equation $u_t + uu_x = u_{xxx}$. The discovery of Kruskal & Zabusky by means of computer simulation that travelling solitary-wave solutions (solitons) interact nonlinearly and emerge unscathed led to a variety of analytical approaches. The equation can be solved exactly but not until a lot is found out about conservation laws for nonlinear equations and the connexion with inverse scattering theory and quantum mechanics. By running backwards some of the arguments used to solve the Korteweg–de Vries equation Lax is able to contribute an elegant chapter on invariant functionals of nonlinear equations of evolution. From the discrete spectrum of linear operators come eigenvalues which are integrals of nonlinear equations of evolution.

Other chapters in the book contain a rather formal theory of quasi-linear hyperbolic systems (C. Dafermos), a theory of collisionless shocks (R. Meyer) and a general study of a combined dissipative dispersive equation (the Burgers–Korteweg–de Vries equation) by the editors.

All the chapters are necessarily brief but they do provide an excellent introduction to the field of nonlinear waves. There is a good bibliography, so that the interested reader can pursue the leads given in the various chapters.

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